INFLUENCE OF TURBULENT Pr NUMBER ON FRICTION AND HEAT TRANSFER AT A PLATE IMMERSED IN A TURBULENT GAS STREAM

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A solution is offered to the problem of determining the friction and heat transfer coefficients for a plate immersed in a turbulent gas stream, using the approximate dependence of heat content on velocity given in reference [1]. The influence of Pr_T on friction and heat transfer is evaluated.

The following relation, between heat content and velocity was established in reference [1] for the case of zero-gradient flow with arbitrary Pr_L and Pr_T:

$$\bar{h} = \bar{h}_w + \left(\frac{\partial \bar{h}}{\partial \varphi}\right)_w S(\varphi) - \bar{u}^2 R(\varphi), \tag{1}$$

where

$$S(\varphi) = \int_0^{\varphi} \exp\left\{-\int_0^{\varphi} \frac{\Pr}{\omega} \frac{\partial \omega (1/\Pr - 1)}{\partial \varphi} d\varphi\right\} d\varphi,$$

$$R(\varphi) = 2\int_0^{\varphi} \exp\left(-\int_0^{\varphi_1} \frac{\Pr}{\omega} \frac{\partial \omega (1/\Pr - 1)}{\partial \varphi} d\varphi\right)\left\{\int_0^{\varphi_1} \Pr\left[\exp\int_0^{\varphi} \frac{\Pr}{\omega} \times \right.\right.$$

$$\left.\left.\times \frac{\partial \omega (1/\Pr - 1)}{\partial \varphi} d\varphi\right] d\varphi\right\} d\varphi_1,$$

$$\omega = \tau/\tau_w, \quad \varphi = v_x/u, \quad \bar{h} = h/H_0, \quad \bar{u}^2 = u^2/2H_0.$$

The present paper makes use of this relation and the principles of the semi-empirical theory of turbulence to evaluate the influence of the Pr_T number on friction and heat transfer at a plate.

Approximate Form of $R(\varphi)$ and $S(\varphi)$ for a Two-Layer System

We shall conventionally divide the complete boundary layer into two zones: a laminar sublayer and a turbulent layer. The dimensionless velocity in these layers will vary, respectively, from 0 to φ_L in the laminar sublayer and from φ_L to l in the turbulent layer. The Pr number will take a value equal to Pr_L in the laminar sublayer and a value equal to Pr_T in the turbulent layer. We then have

$$S(\varphi) = S(\varphi_L) + \frac{1}{\Pr_L}\int_{\varphi_L}^{\varphi} \Pr_T\left(\exp - \int_0^{\varphi} \frac{1-\Pr}{\omega} \frac{\partial \omega}{\partial \varphi} d\varphi\right) d\varphi_1;$$

$$R(\varphi) = R(\varphi_L) + 2\int_{\varphi_L}^{\varphi} \Pr_T\left(\exp - \int_0^{\varphi_1} \frac{1-\Pr}{\omega} \frac{\partial \omega}{\partial \varphi} d\varphi\right) \times$$

$$\times \left[\int_0^{\varphi}\left(\exp\int_0^{\varphi} \frac{1-\Pr}{\omega} \frac{\partial \omega}{\partial \varphi} d\varphi\right) d\varphi\right] d\varphi_1.$$

We represent $\omega(\varphi)$ approximately in the form of a third-degree polynomial:

$$\omega = 1 - \varphi^3.$$

Assuming that Pr_T and Pr_L are constants, we calculate $S(\varphi_L)$ and $R(\varphi_L)$. Making a numerical approximation to the results obtained, we have

$$S(\varphi_L) = \varphi_L, \quad R(\varphi_L) \approx \mathrm{Pr}_L \varphi_L^2.$$

The integrals $S(\varphi)$ and $R(\varphi)$, with account for the form of $S(\varphi_L)$ and $R(\varphi_L)$, become

$$S(\varphi) = \varphi_L\left[1 - \frac{\mathrm{Pr}_T}{\mathrm{Pr}_L}\,\omega_L^{\mathrm{Pr}_L - \mathrm{Pr}_T}\right] + \frac{\mathrm{Pr}_T}{\mathrm{Pr}_L}\,\omega_L^{\mathrm{Pr}_L - \mathrm{Pr}_T} I_1,$$

$$R(\varphi) = \varphi_L^2(\mathrm{Pr}_L - \mathrm{Pr}_T) + 2\mathrm{Pr}_T I_2, \tag{2}$$

where

$$I_1 = \int_0^\varphi \omega^{\mathrm{Pr}_T - 1}\, d\varphi, \quad I_2 = \int_0^\varphi \omega^{\mathrm{Pr}_T - 1}\left(\int_0^\varphi \omega^{1-\mathrm{Pr}_T}\, d\varphi\right) d\varphi.$$

Calculation of I_1 and I_2 for Pr_T varying from 0.5 to 2 shows that I_1 and I_2 may be put in the approximate form

$$I_1 = a_1 \varphi, \quad I_2 = a_2 \varphi^2, \tag{3}$$

where

$$a_1 = 1.214 - 0.214\,\mathrm{Pr}_T, \quad a_2 = 0.65 - 0.15\,\mathrm{Pr}_T.$$

As regards $I_1(1)$ and $I_2(1)$, $I_1(1)$ may be accurately expressed in terms of gamma functions, and $I_2(1)$ approximately, as follows:

$$I_1(1) = \frac{1}{3}\,\frac{\Gamma(^1/_3)\,\Gamma(\mathrm{Pr}_T)}{\Gamma(\mathrm{Pr}_T + {}^1/_3)},$$

$$I_2(1) \simeq \frac{1}{3}\,\frac{\Gamma(^2/_3)\,\Gamma(\mathrm{Pr}_T)}{\Gamma(\mathrm{Pr}_T + {}^2/_3)} - \frac{1-\mathrm{Pr}_T}{12}\,\frac{\Gamma(^5/_3)\,\Gamma(\mathrm{Pr}_T)}{\Gamma(^5/_3 + \mathrm{Pr}_T)}.$$

Determining $(\partial h/\partial\varphi)_w$ from the boundary conditions $h = h_0$ when $\varphi = 0$, we obtain

$$\left(\frac{\partial \bar{h}}{\partial \varphi}\right)_w = \frac{\bar{h}_0 + \bar{u}^2 R(1) - \bar{h}_w}{S(1)}, \tag{4}$$

where

$$S(1) = \alpha\varphi_L + \beta I_1(1),$$

$$R(1) = \gamma\varphi_L^2 + 2\,\mathrm{Pr}_T I_2(1),$$

$$\alpha = 1 - \frac{\mathrm{Pr}_T}{\mathrm{Pr}_L}\,\omega_L^{\mathrm{Pr}_L - \mathrm{Pr}_T}; \quad \beta = \frac{\mathrm{Pr}_T}{\mathrm{Pr}_L}\,\omega_L^{\mathrm{Pr}_L - \mathrm{Pr}_T},$$

$$\gamma = \mathrm{Pr}_L - \mathrm{Pr}_T.$$

Substituting (2) into (1) and taking (3) into account, we can approximate to the dependence of h on φ in the turbulent layer by means of a second-degree polynomial in φ:

$$\bar{h} = A\varphi^2 + B\varphi + C, \tag{5}$$

where

$$A = -2\,\mathrm{Pr}_T a_2 \bar{u}^2,$$

$$B = (1 - \bar{h}_w + [R(1) - 1]\bar{u}^2)\,\beta a_1/S(1),$$

$$C = \bar{h}_w + (1 - \bar{h}_w + [R(1) - 1]\bar{u}^2)\,\alpha\varphi_L/S(1) - \bar{u}^2\gamma\varphi_L.$$

In the laminar sublayer

$$S(\varphi) \approx \varphi, \quad R(\varphi) = \mathrm{Pr}_L \varphi^2,$$

$$\bar{h} = \bar{h}_w + \left(\frac{\partial \bar{h}}{\partial \varphi}\right)_w \varphi - \bar{u}^2\,\mathrm{Pr}_L \varphi^2 = \bar{h}_w + B'\varphi + A'\varphi^2. \tag{6}$$

Derivation of the Velocity Profile

We shall determine the velocity distribution law over the boundary layer.

In accordance with the basic assumptions of the semi-empirical theory of turbulence, the friction stress is given by:

$$y \geqslant \delta_L, \quad \tau = \rho k^2 y^2 u^2 (\partial \varphi/\partial y)^2,$$
$$y \leqslant \delta_L, \quad \tau = \mu u \partial\varphi/\partial y.$$

In accordance with the preceding paragraph, we make the assumption

$$\tau/\tau_w = 1 - \varphi^3.$$

We shall further suppose that

$$\rho/\rho_w = h_w/h = \bar{h}_w/\bar{h}.$$

Using the expressions derived, after substituting $\bar{h}$ from (5), for determining the dependence $\varphi(y)$ we obtain

$$y \leqslant \delta_L, \quad \mu u \frac{\partial \varphi}{\partial y} = \tau_w (1 - \varphi^3); \tag{7}$$

$$y \geqslant \delta_L, \quad \frac{\tau_w}{\rho_w u^2} \frac{1}{k^2 y^2} = \frac{\bar{h}_w (\partial \varphi/\partial y)^2}{(1-\varphi^3)(A\varphi^2 + B\varphi + C)}. \tag{8}$$

Integrating (8) approximately, and taking into account that $A < 0$, on condition that $\varphi = 1$ when $y = \delta$, we obtain

$$\ln \frac{y}{\delta} = \sqrt{\bar{h}_w}\, k \zeta \frac{1{,}107}{\sqrt{-A}} \left(\arcsin \frac{-2A\varphi - B}{\sqrt{B^2 - 4AC}} - \arcsin \frac{-2A - B}{\sqrt{B^2 - 4AC}} \right). \tag{9}$$

In the laminar sublayer, assuming that $\mu = \mu_w (h/h_w)^n$, after substituting the value of h from (6) into (7), for determining $\varphi(y)$ we obtain the equation

$$\left(1 + \frac{B'}{\bar{h}_w}\varphi + \frac{A'}{\bar{h}_w}\varphi^2\right) \frac{\partial \varphi}{\partial y} \approx \frac{\tau_w}{\mu_w u}, \tag{10}$$

whose solution may be written approximately as:

$$\varphi \left(1 + \frac{nB'}{\bar{h}_w}\varphi + \frac{nA'}{\bar{h}_w}\varphi^2\right) = \frac{\tau_w}{\mu_w u} y.$$

This formula is valid for $0 \leq y \leq \delta_L$.

Determination of the Thickness of the Laminar Sublayer and the Flow Velocity at its Edge

The velocity derivative has a discontinuity at the edge of the laminar sublayer

$$\left(\frac{\partial v_x}{\partial y}\right)_{y=\delta_L - 0} = k_1 \left(\frac{\partial v_x}{\partial y}\right)_{y=\delta_L + 0}. \tag{11}$$

From the relations of the semi-empirical theory, $\mu \frac{\partial v_x}{\partial y} = \tau$ when $y \leqslant \delta_L$, $\tau = \rho l^2 \left(\frac{\partial v_x}{\partial y}\right)^2$, and when $y \geq \delta_L$, assuming $l = ky$, we find

$$\left(\frac{\partial v_x}{\partial y}\right)_{y=\delta_L - 0} = \frac{\tau_L}{\mu_L}; \quad \left(\frac{\partial v_x}{\partial y}\right)_{y=\delta_L + 0} = \frac{1}{k\delta_L}\sqrt{\frac{\tau_L}{\rho_L}}.$$

Substituting into (11), we obtain

$$\delta_L = \frac{k_1}{k} \frac{\mu_L}{\sqrt{\rho_L \tau_L}}. \tag{12}$$

Since δ_L/δ is small, we may put $\tau_L \approx \tau_W$. Taking into account that $\rho/\rho_W = h_W/h$, $\mu = \mu_W (h/h_W)^n$ we obtain

$$\frac{\mu_L}{\sqrt{\rho_L}} = \frac{\mu_w}{\sqrt{\rho_w}} \left[1 + \left(n + \frac{1}{2}\right)\frac{B'}{\bar{h}_w}\varphi_L + \left(n + \frac{1}{2}\right)\frac{A'}{\bar{h}_w}\varphi_L^2\right].$$

Substituting $\mu_L/\sqrt{\rho_L}$ into (12), we obtain

$$\delta_L = \frac{k_1}{k} \frac{\mu_w}{\rho_w \sqrt{\tau_w/\rho_w}} \left[1 + \left(n + \frac{1}{2}\right) \frac{B'}{\bar{h}_w} \varphi_L + \left(n + \frac{1}{2}\right) \frac{A'}{\bar{h}_w} \varphi_L^2\right]. \quad (13)$$

To determine φ_L, we put $y = \delta_L$, $\varphi = \varphi_L$ in (10), and obtain

$$\varphi_L\left(1 + \frac{n}{2} \frac{B'}{\bar{h}_w} \varphi_L + \frac{n}{3} \frac{A'}{\bar{h}_w} \varphi_L^2\right) =$$

$$= \frac{k_1}{k} \sqrt{\frac{\tau_w}{\rho_w}} \frac{1}{u} \left[1 + \left(n + \frac{1}{2}\right) \frac{B}{\bar{h}_w} \varphi_L + \left(n + \frac{1}{2}\right) \frac{A'}{\bar{h}_w} \varphi_L^2\right]. \quad (14)$$

This equation may be solved by the method of successive approximations. We take as the first approximation

$$\varphi_L = \frac{k_1}{k} \frac{v_*}{u} = \frac{k_1}{k\zeta}, \quad \delta_L = \frac{k_1}{k} \frac{\nu_w \zeta}{u},$$

where

$$\zeta = \frac{u}{\sqrt{\tau_w/\rho_w}} = \frac{u}{v_*}.$$

Deviation of the Relation Between ζ and δ

Putting $y = \delta_L$, $\varphi = \varphi_L$ in (9), we obtain the following equation relating ζ and δ:

$$\ln \frac{\delta_L}{\delta} = \frac{1.107}{\sqrt{-A}} k\zeta \sqrt{\bar{h}_w} \left(\arcsin \frac{-2A\varphi_L - B}{\sqrt{B^2 - 4AC}} - \arcsin \frac{-2A - B}{\sqrt{B^2 - 4AC}}\right). \quad (15)$$

Replacing δ_L and φ_L by their values in the first approximation, taking into account that $1/k\zeta$ is small, expanding the terms in brackets in series, and retaining terms containing $1/k\zeta$ in the first degree, we obtain

$$u\delta/\nu_w = Dk\zeta \exp(Ck\zeta/u).$$

Here

$$C = 1{,}107 \sqrt{\frac{\bar{h}_w}{2\mathrm{Pr}_т a_2}} \left\{\arcsin \frac{1 - \bar{h}_w + l\bar{u}^2}{\varepsilon} - \arcsin \frac{1 - \bar{h}_w + f\bar{u}^2}{\varepsilon}\right\};$$

$$l = 2\mathrm{Pr}_т I_2(1) - 1; \quad f = l - 4\mathrm{Pr}_т a_2 I_1(1) a_1;$$

$$\varepsilon^2 = (1 - \bar{h}_w + l\bar{u}^2)^2 + 8\mathrm{Pr}_т a_2 \bar{u}^2 \bar{h}_w I_1^2(1)/a_1^2;$$

$$D = \frac{k_1}{k^2} \exp\left\{- 1.107k_1 \left(1 - \frac{\mathrm{Pr}_т - \mathrm{Pr}_л}{\varepsilon^2 a_1 \mathrm{Pr}_т} \times\right.\right.$$

$$\times \left(1 - \bar{h}_w + l\bar{u}^2\right)\left[1 + \bar{h}_w + l\bar{u}^2 - 2\left\{\frac{\bar{h}_w}{2}\left(1 - \frac{a_1}{I_1}\right) + \frac{1}{2}\left(1 + \frac{a_1}{I_1}\right) - \right.\right.$$

$$\left.\left.\left. - \bar{u}^2\left[2\mathrm{Pr}_т a_2 \frac{I_1}{a_1} - \frac{1}{2}\left(\frac{a_1}{I_1} + 1\right) l\right]\right\} \frac{\sqrt{\bar{h}_w}}{\sqrt{1 - \bar{u}^2}}\right]\right)\right\}.$$

In order to obtain the friction drag of the plate, we must find a second equation relating the friction and the boundary layer thickness.

We obtain this equation by using the integral relation expressing the momentum law.

Determination of the Ratio δ^{**}/δ

By definition

$$\frac{\delta^{**}}{\delta} = \int_0^1 \frac{\rho}{\rho_0} \varphi(1 - \varphi) d\frac{y}{\delta} = \frac{\rho_w}{\rho_0} I.$$

Since $\delta_L/\delta \ll 1$, in determining δ^{**}/δ we shall take values of φ and ρ/ρ_w, for the turbulent regime. To calculate δ^{**}/δ, we integrate successively by parts and put the result in the form of a series in $1/k\zeta$:

$$I = \int_0^1 \frac{\rho}{\rho_w}(\varphi - \varphi^2)\, d\,\frac{y}{\delta} = \frac{\sqrt{\bar{h}_w}}{1.107k\zeta}\,\frac{1}{\sqrt{1-\bar{u}^2}} + \frac{F_1}{k\zeta^2} + \frac{F_2}{k\zeta^3} + \ldots\,.$$

Then, taking account of the expression for ρ_w/ρ_0, we obtain

$$\frac{\delta^{**}}{\delta} = \sqrt{\frac{1-\bar{u}^2}{\bar{h}_w}}\,\frac{1}{1.107k\zeta} + \ldots\,.$$

Determination of the Friction Law

We use the integral relation expressing the momentum law

$$\frac{d}{dx}\,\rho_0 u^2 \delta^{**} + \rho_0 u \frac{du}{dx}\,\delta^{*} = \tau_w\,.$$

For the plate u = const and ρ_0 = const, and therefore ρ_w = const also; making the assumption that T_w = const, we may write this equation as

$$\frac{d}{dx}\left(\frac{u\delta}{\nu_w} I\right) = \frac{u}{\nu_w}\,\frac{\tau_w}{\rho_w u^2} = \frac{1}{\zeta^2}\,\frac{u}{\nu_w}\,.$$

Replacing I and $u\delta/\nu_w$ by their values, evaluating the last integral approximately, and taking into account that $1/k\zeta$ is small, we obtain

$$\frac{ux}{\nu_w} = D\zeta^2 \exp\frac{C}{\bar{u}}k\zeta\sqrt{\frac{\bar{h}_w}{1-\bar{u}^2}}\left(\frac{1}{1.107} + \frac{F_1}{k\zeta} + \frac{F_2}{k\zeta^2} + \ldots\right). \qquad (16)$$

The local friction coefficient is

$$c_f = \frac{2\tau_w}{\rho_0 u^2} = \frac{2}{\zeta^2}\,\frac{1-\bar{u}^2}{\bar{h}_w}\,.$$

The friction coefficient for a plate of length l will be

$$C_f = \int_0^l \frac{c_f}{l}\,dx = \frac{2\delta^{**}_{x=l}}{l} = \frac{\nu_w}{u}\left(\frac{u\delta}{\nu_w} I\right)_{x=l}\frac{2}{l}\,\frac{1-\bar{u}^2}{\bar{h}_w}\,.$$

Substituting for $u\delta/\nu_w$ and I, we obtain

$$C_f = \frac{2}{1.107}\sqrt{\frac{1-\bar{u}^2}{\bar{h}_w}}\,\frac{\nu_w}{ul}\,D\exp\frac{C}{\bar{u}}k\zeta_l\,, \qquad (17)$$

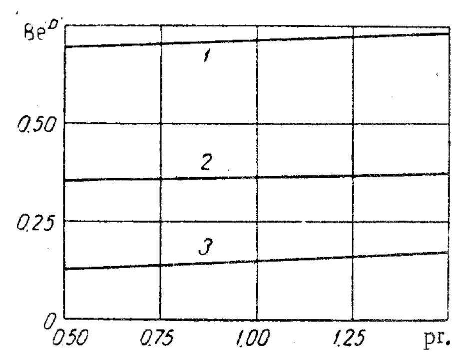

Fig. 1. Dependence of Be^D on Pr_T for h_w = 1 and Pr_L = 1; 1–M = 2; 2–5; 3–10.

where we determine ζ_l from (16), putting $x = l$. For an incompressible fluid C_f is obtained for $\bar{u} = 0$, $\bar{h}_w = 1$, $\nu_w = \nu_0$.

Interpolation Formula for C_f

The formulas obtained for c_f and C_f may be simplified within a certain range of variation of ζ. When ζ varies in the limits 8 to 20, as shown in [2], the function $\ln(\zeta^2 \exp \zeta)$ may be represented approximately as the straight line:

$$\ln(\zeta^2 \exp\zeta) = n_1 + n_2\zeta,$$

where n_1 = 2.9; n_2 = 1.16 for the range of variation of ζ indicated. In this case

$$\left[\exp\frac{C}{\bar{u}}k\zeta\right]k^2\zeta^2 = \frac{u^2}{C^2}\exp(n_1 + n_2\bar{\zeta}), \quad \bar{\zeta} = Ck\zeta/\bar{u}.$$

Formula (16) may be written in the form

$$\frac{ux}{\nu_w} = \frac{D\bar{u}^2}{k^2 C^2} \exp(n_1 + n_2 \bar{\zeta}) \left(\sqrt{\frac{\bar{h}_w}{1 - \bar{u}^2}} \frac{1}{1.107} + \ldots \right). \tag{18}$$

Putting x = l and neglecting the terms $\frac{F_1}{k\zeta} + \frac{F_2}{(k\zeta)^2}$ in comparison with the first, we obtain

$$n_1 + n_2 \bar{\zeta}_l = \ln \left(\frac{k^2 C^2}{D\bar{u}^2} \frac{ul}{\nu_w} \sqrt{\frac{1 - \bar{u}^2}{\bar{h}_w}} 1.107 \right).$$

Substituting ζ_1 from this formula into (17), we obtain

$$C_f = (1.107)^{\frac{3-n_2}{n_2}} \left(\frac{ul}{\nu_0} \right)^{\frac{1-n_2}{n_2}} D^{\frac{n_2-1}{n_2}} \left(\frac{c}{\bar{u}} \right)^{\frac{2}{n_2}} (\bar{h}_w)^{\frac{n_2-1}{2n_2}} (1 - \bar{u}^2)^{\frac{3-n_2}{2n_2}} \times$$

$$\times \left(\frac{\mu_w}{\mu_0} \right)^{\frac{n_2-1}{n_2}} 2k^{\frac{2}{n_2}} \exp\left(-\frac{n_1}{n_2} \right) (2\,\mathrm{Pr}_T a_2)^{-\frac{1}{n_2}}.$$

Substituting the value of C and D and taking into account that $n_1 = 2.9$; $n_2 = 1.16$; $k = 0.39$; $k_1/k = 11$, we obtain

$$C_f = 0.027\,\mathrm{Re}^{-0.139} B(\bar{u}, \bar{h}_w) \exp D',$$

$$B = (\bar{h}_w)^{0.0695} (1 - \bar{u}^2)^{0.79} \left(\frac{\mu_w}{\mu_0} \right)^{0.139} (2\mathrm{Pr}_T a_2)^{-0.861} \times$$

$$\times \left[\frac{1}{\bar{u}} \left(\arcsin \frac{\bar{h}_w - 1 - f\bar{u}^2}{\varepsilon} - \arcsin \frac{\bar{h}_w - 1 - l\bar{u}^2}{\varepsilon} \right) \right]^{1.722},$$

$$D' = 0.139 \cdot 1.107 k_1 \frac{\mathrm{Pr}_T - \mathrm{Pr}_Л}{\varepsilon^2 \mathrm{Pr}_T a_1} (1 - \bar{h}_w + l\bar{u}^2) \left[1 + \bar{h}_w + l\bar{u}^2 - 2 \left(\frac{\bar{h}_w}{2} \left(1 - \frac{a_1}{I_1} \right) + \right. \right.$$

$$\left. \left. + \frac{1}{2} \left(1 + \frac{a_1}{I_1} \right) - \bar{u}^2 \left(2\mathrm{Pr}_T a_2 \frac{I_1}{a_1} - \frac{1}{2} \left(\frac{a_1}{I_1} + 1 \right) l \right) \right) \sqrt{\frac{\bar{h}_w}{1 - \bar{u}^2}} \right].$$

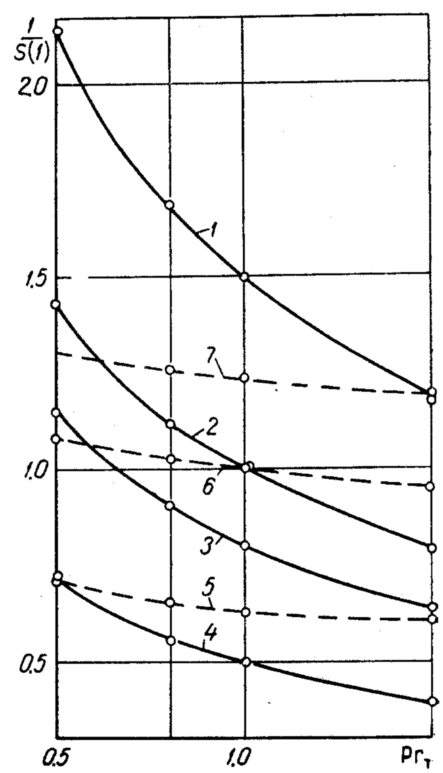

Fig. 2. Dependence of 1/S(1) on Pr_T and Pr_L for $\varphi_L = 0$ (1 - Pr_L = 1.5; 2 - 1.0; 3 - 0.8; 4 - 0.5) and for $\varphi_T = 0.5$ (5 - Pr_L = 0.5; 6 - 1; 7 - 1.5).

The factor $B(\bar{u}, \bar{h}_w) \exp D'$ characterizes the influence of the variable heat content and compressibility on the friction drag of the plate. The factor $\exp D'$ characterizes the influence of Pr_L on the value of C_f. When $Pr_T = 1$, we obtain the formula given in [2] for C_f.

To evaluate the influence of Pr_T on the friction drag, we give the dependence of $Be^{D'}$ on Pr_T for $\bar{h}_w = 1$ and for various M (Fig. 1). It may be seen from the graph that the influence of Pr_T on C_f is insignificant. The influence of Pr_T on q_w is more substantial, as may be seen from the expression for S(1) and R(1).

Influence of Pr_T on Heat Transfer

Using the expression for $h(\varphi)$, derived at the beginning of this article, we obtain that

$$q_w = \frac{\lambda_w}{C_{p_w}} \frac{\partial h}{\partial y}\bigg|_{y=0} = \frac{\lambda_w}{C_{p_w}} \left(\frac{\partial h}{\partial \varphi}\right)_w \frac{1}{u} \frac{\partial v_x}{\partial y}\bigg|_{y=0}.$$

Substituting the value of $\left(\frac{\partial h}{\partial \varphi}\right)_w$ from (4), and taking into consideration that $\frac{\partial v_x}{\partial y}\bigg|_{y=0} = \frac{\tau_w}{\mu_w}$, we have

$$q_w = \frac{1}{\Pr(0)} \frac{\tau_w}{u} \frac{h_r - h_w}{S(1)},$$

where

$$h_r = h_0 + R(1)\, u^2/2.$$

For the dimensionless heat transfer coefficient we obtain the expression

$$\mathrm{St} = \frac{q_w}{(h_r - h_w)\rho_0 u} = \frac{c_f}{2\Pr(0)} \frac{1}{S(1)}.$$

Figure 2 shows the dependence of $1/S(1)$ as a function of Pr_L and Pr_T for two values of φ_L. It follows from the graphs in this figure that Pr_T has an appreciable influence on the value of the St number.

NOTATION

v_x, v_y—velocity components along the coordinate axes; p, ρ, μ, λ—pressure, viscosity and thermal conductivity respectively; h—enthalpy of unit mass of gas; $H = h + v_x^2/2$—total heat content of unit mass of gas; τ and q_y—components of friction stress tensor and heat flux vector; R—gas constant; M—molecular weight; Pr—Prandtl number; u, ρ_0, h_0, $H_0 = h + u^2/2$—values characterizing the external flow; ρ_w—density at wall; δ_L—thickness of laminar sublayer; φ_L—velocity at edge of laminar sublayer.

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